The Degrees of Freedom of the K-pair-user Full-Duplex Two-way Interference Channel with a MIMO Relay

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***Abstract*—In a K-pair-user two-way interference channel (TWIC), $2K$ messages and $2K$ transmitters/receivers form a K-user IC in the forward direction (K messages) and another K-user IC in the backward direction which operate in full-duplex. All nodes may interact, or adapt inputs to past received signals. The optimal degrees of freedom (DoF, also known as the multiplexing gain) is known to be K [1]: full-duplex operation doubles the DoF, but interaction does not further increase the DoF. In this paper, we characterize the DoF of the K-pair-user TWIC with a MIMO, full-duplex relay. If the relay is non-causal/instantaneous (at time k forwards a function of its received signals up to time k) and has $2K$ antennas, we demonstrate a one-shot scheme where the relay mitigates all interference to achieve the interference-free $2K$ DoF. In contrast, if the relay is causal (at time k forwards a function of its received signals up to time $k-1$), we show that a full-duplex MIMO relay cannot increase the DoF of the K-pair-user TWIC beyond K, as if no relay or interaction is present.**

I. Introduction

In wireless communications, full-duplex (in-band) operation enables real two-way or bidirectional communications. Although the current two-way systems operate as two one-way communications employing either time or frequency division, recently, the design of full-duplex (in-band) wireless systems [2], [3] shows great promise for increasing data rates in future wireless technologies.

We study the impact of full-duplex operation to *two-way networks with interference*. In particular, the two-way interference channel (TWIC) has been studied in [4]–[7]. More recently, a natural extension of the TWIC, the K-pair-user full-duplex TWIC has been considered in [1], [8], in which there are $2K$ independent messages: K-messages to be transmitted over a K-user interference channel (IC) in the $\rightarrow$ direction simultaneously with K-messages to be transmitted over an in-band K-user IC in the $\leftarrow$ direction. All $2K$ nodes in this network act as both sources and destinations of messages. This allows for *interaction* between the nodes: a node's channel inputs may be functions of its message and previously received signals. The degrees of freedom (DoF) [9] of the K-pair-user TWIC has been shown to be K [1], i.e. $K/2$ in each direction. This demonstrates that interaction between users does *not* increase the DoF of the K-pair-user TWIC beyond the doubling that full-duplex operation yields, since the DoF of the one-way K-user IC is $K/2$ [10], [11].

Relays are additional nodes which do not have messages of their own and may aid the other nodes in transmitting their signals. We ask how much a relay may increase the DoF of the K-pair-user full-duplex TWIC. Interestingly, we show that the presence of a non causal or instantaneous relay with sufficient number of antennas may increase the DoF to the maximal value of $2K$. That is, in a DoF sense, a non causal MIMO relay may cancel out all the interference in the K-pair-user TWIC in both directions simultaneously. In sharp contrast, we show that if the MIMO relay is causal, then regardless of the number of antennas, it cannot increase the DoF of the K-pair-user TWIC, i.e. the DoF remains K.

The DoF of many one-way communication networks have been characterized, however, much less is known about the DoF of two-way communications. Recently, [12] considered a *half-duplex* two-pair two-way interference channel (without interaction) with a 2-antenna relay and showed that 4/3 DoF are achievable. No converse results were provided. In [13], the authors identified the DoF of the full-duplex 2-pair and 3-pair two-way multi-antenna relay MIMO interference channel, in which users only communicate through the relay (no direct links). Authors in [14] showed that for almost all constant channel coefficients of fully connected two-hop wireless networks with K sources, K relays and K destinations (source nodes are not destination nodes as they are here, i.e. the network is one-way), the DoF is K.

Our work differs from prior work in that we consider an *interactive*, *full-duplex* Gaussian K-pair-user TWIC with a MIMO relay (non-causal or causal) for the first time, and obtain exact DoF results (achievability and converse match).

II. System Model

We consider a K-pair-user two-way interference channel with a MIMO relay as shown in Fig. 1, where there are $2K$ messages and $2K$ terminals forming a K-user IC in the $\rightarrow$ direction (K messages) and another K-user IC in the $\leftarrow$ direction (K messages). A MIMO relay connects to all $2K$ terminals and helps in communicating messages and managing interference in the network. All nodes operate in full-duplex mode and can transmit and receive signals simultaneously.

The relay has M antennas and operates either in a non-causal or "instantaneous" fashion, or in a causal fashion.

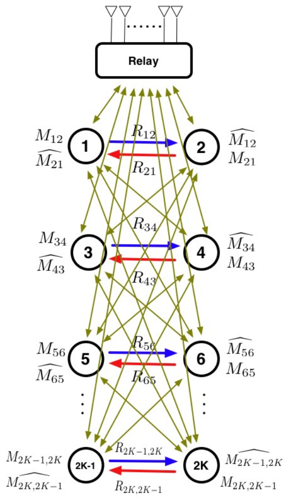

Fig. 1. K-pair-user two-way interference channel with a MIMO relay. M_{ij} denotes the message known at node i and desired at node j; $\widehat{M_{ij}}$ denotes that j would like to decode the message M_{ij} from node i.

By "instantaneous" (non causal, relay-without-delay [15]) we refer to its ability to decode and forward signals received at the previous and *current* (but *not* future) time slots.This requirement is significantly less strict than a *cognitive* relay, which would know all users' signals prior to transmission. Here messages are obtained over the air; the only idealization is the non causality or access to received signals from the current time slot. Mathematically, we may describe non causal and causal relaying functions, for each $k = 1, 2, \cdots n$, as

Non-causal / instantaneous relaying:

$$\mathbf{X}_R[k] = g_k(\mathbf{Y}_R[1], \mathbf{Y}_R[2], ..., \mathbf{Y}_R[k])$$

Causal relaying:

$$\mathbf{X}_R[k] = g_k(\mathbf{Y}_R[1], \mathbf{Y}_R[2], \cdots \mathbf{Y}_R[k-1]),$$

where $\mathbf{X}_R[k]$ is a $M \times 1$ (M antennas) vector signal transmitted by the relay at time slot k; $g_k()$ is a deterministic function; and $\mathbf{Y}_R[l], l \in \{1, 2, ..., k\}$ is the $M \times 1$ vector of signals received at the relay at time slot l. The relay is subject to per symbol transmit power constraints over all antennas $E[||\mathbf{X}_R[k]||^2] \le P_R$, $\forall k \in \{1, 2, \cdots n\}$, and global channel state information knowledge is assumed at all nodes. At each time slot k, the system input/output relationships are:

$$Y_p[k] = \sum_{m=1}^{K} h_{2m,p}[k] X_{2m}[k] + \mathbf{h}^*_{Rp}[k]\mathbf{X}_R[k] + Z_p[k], \quad p = 1, 3, ..., 2K-1 \tag{1}$$

$$Y_q[k] = \sum_{m=1}^{K} h_{2m-1,q}[k] X_{2m-1}[k] + \mathbf{h}^*_{Rq}[k]\mathbf{X}_R[k] + Z_q[k], \quad q = 2, 4, ..., 2K \tag{2}$$

$$\mathbf{Y}_R[k] = \sum_{m=1}^{2K} \mathbf{h}_{m,R}[k] X_m[k] + \mathbf{Z}_R[k] \tag{3}$$

where $X_l[k], Y_l[k], l \in \{1, 2, ..., 2K\}$ are the inputs and outputs of user l at time slot k, and $h_{ij}[k], i, j \in \{1, 2, ..., 2K\}$ is the channel coefficient from node i to node j at time slot k. The network is subject to complex Gaussian noise $Z_l[k] \sim \mathcal{CN}(0, 1), l \in \{1, 2, ..., 2K\}$ which are independent across users and time slots. In addition, $\mathbf{h}_{ij}[k], i, j \in \{1, 2, ..., 2K, R\}$ is the $M \times 1$-dimensional channel coefficient vector from node i to node j at time slot k (i or j must be the relay node R), and $\mathbf{Z}_R[k] \sim \mathcal{CN}(\mathbf{0}, \mathbf{I})$ is the complex Gaussian noise vector at the relay. The terms in bold represent vectors (due to the MIMO relay). We use * to denote conjugate transpose and ^{T} to denote transpose.

We consider time-varying channel coefficients, which for each channel use are all drawn from a continuous distribution and whose absolute values are bounded between a nonzero minimum value and a finite maximum value. Note one can also alternatively consider a frequency selective system model.

We further assume per user, per symbol power constraints $E[|X_i[k]|^2] \le P, i \in \{1, 2, ..., 2K\},\ k \in \{1, 2, \cdots n\}$, for block length n. User $2i-1$ and $2i$ wish to exchange messages for $i = 1, 2, \cdots K$ (user 1 sends to 2, 2 to 1,..., 2K-1 to 2K, 2K to 2K-1) with *interactive* encoding functions

$$X_i[k] = f(M_{ij}, Y_i^{k-1}), \quad k = 1, 2, \cdots n$$

at rate $R_{i,j} = \frac{\log_2 |M_{ij}|}{n}$, where Y_i^{k-1} denotes the vector $(Y_i[1], \cdots Y_i[k-1])$ from time slot, or channel use 1 to $k-1$ received at user i. In other words, all users in this network can adapt current channel inputs to previously received channel outputs. The nodes $2i-1$ and $2i$ have decoding functions which map $(Y^n_{2i-1}, M_{2i-1,\, 2i})$ to an estimate of $M_{2i,\, 2i-1}$ and $(Y^n_{2i}, M_{2i,\, 2i-1})$ to $M_{2i-1,\, 2i}$, respectively. A rate tuple $(R_{i,i+1}(P), R_{i+1,i}(P))_{i \in \{1,3,...,2K-1\}}$, where we use the argument P simply to remind the reader that this rate is indeed a function of the power constraint P, is said to be achievable if there exist a set of interactive encoders and decoders such that the desired messages may be estimated with arbitrarily small probability of error when the number of channel uses n tends to infinity. The sum DoF characterizes the sum capacity of this Gaussian channel at high SNR and is defined as the maximum over all achievable $(R_{i,i+1}(P), R_{i+1,i}(P))_{i \in \{1,3,...,2K-1\}}$ of

$$\begin{aligned} d_{sum} &= \sum_{i=1,3,...,2K-1} (d_{i,i+1} + d_{i+1,i}) \\ &= \limsup_{P \to \infty} \frac{\sum_{i=1,3,...,2K-1} (R_{i,i+1}(P) + R_{i+1,i}(P))}{\log(P)}. \end{aligned}$$

Notice the implicit definitions of the DoF of the link from user i to user $i+1$, $d_{i,i+1}$ and the reverse $d_{i+1,i}$.

The received signal at any given node may be broken down into four types of signals: the self-interference signal (SI, sent by itself, known to itself); the interference signal (sent by the undesired user(s) from the opposite side); the desired signal (sent by the desired user); the undesired signal (sent by the undesired user(s) on same side), respectively. E.g., at receiver 1, s_{12} is a self-interference signal (SI); $s_{43}, s_{65}, \cdots, s_{2K,2K-1}$ are interference signals; s_{21} is the desired signal, and $s_{34}, s_{56}, \cdots, s_{2K-1,2K}$ are undesired signals.

Note we have already removed self-interference signals from the input/output equations (1)-(2), but SI terms may still be transmitted by the relay (or other users due to adaptation) and hence received.

III. DoF of K-pair-user two-way IC with an instantaneous MIMO relay

In this section, we investigate the DoF of the K-pair-user two-way IC with an instantaneous MIMO relay with $M = 2K$ antennas in the system model described in Section II. Remarks on how to reduce the number of antennas at the relay, at the expense of for example diminished achievable degrees of freedom, or requiring partial cognition of the messages at the relay, can be found in [1]. The main result of this section is stated in the following theorem:

Theorem 1: The full-duplex K-pair-user two-way interference channel, with interaction and with an instantaneous $2K$-antenna relay has $2K$ degrees of freedom.

Proof:

1) Converse: The converse is trivial since for a $2K$-user, $2K$ message unicast network where all sources and destinations have a single antenna, the maximum degrees of freedom cannot exceed $2K$ by cut-set arguments, even with adaptation/interaction at all nodes.

2) Achievability: We propose a simple "one-shot" scheme that achieves $2K$ DoF for the K-pair-user two-way IC with the help of an instantaneous $2K$-antenna relay. We consider the Gaussian channel model at high SNR, and hence noise terms are ignored from now on.

The $2K$ users each transmit a symbol s_{ij} (from user i to intended user j) and the relay receives:

$$\mathbf{Y}_R = \sum_{i=1}^{2K} \mathbf{h}_{i,R} s_{ij}, \text{ for the appropriate } j \text{ values, see Fig. 1.}$$

The $2K$-antenna relay (with global CSI) decodes all $2K$ symbols using a zero-forcing decoder [17], and due to the instantaneous property, transmits the following signal in the same time slot: l

$$\mathbf{X}_R = \sum_{i=1}^{2K} \mathbf{u}_{ij} s_{ij}$$

where $\mathbf{u}_{ij}$ denote the $2K \times 1$ beamforming vectors carrying signals from user i to intended user j. Now at receiver 1 (for example),

$$Y_1 = \sum_{m=1}^{K} h_{2m,1} s_{2m,2m-1} + \mathbf{h}_{R1}^* \mathbf{X}_R. \tag{4}$$

To prevent undesired signals from reaching receiver 1, the relay picks beamforming vectors such that

$$\mathbf{u}_{ij} \in null(\mathbf{h}_{R1}^*), \quad i = 3, 5, ..., 2K-1, \ j \text{ as appropriate}, \tag{5}$$

where $null(\mathbf{A})$ denotes the null space of $\mathbf{A}$. Since there are $2K$ antennas at the relay, $null(\mathbf{h}_{R1}^*)$ has dimension $2K-1$.

At receiver 1, the interference signals received from the relay are used to neutralize the interference signals received from the transmitters. To do this, we design the beamforming vectors to satisfy:

$$h_{2m,1} + \mathbf{h}_{R1}^* \mathbf{u}_{2m,2m-1} = 0, \quad m = 2, 3, ..., K. \tag{6}$$

The $2K \times 1$ beamforming vectors satisfying the needed constraints always exist, by a dimensionality argument, along with the random channel coefficients. To see this, take $\mathbf{u}_{34}$ as an example. We wish to construct $\mathbf{u}_{34}$ such that the following conditions are satisfied:

$$\mathbf{u}_{34} \in null(\mathbf{h}_{Rp}^*), \quad p = 1, 5, 7, ..., 2K-1 \tag{7}$$

$$h_{3q} + \mathbf{h}_{Rq}^* \mathbf{u}_{34} = 0, \quad q = 2, 6, 8, ..., 2K. \tag{8}$$

From $\mathbf{u}_{34} \in null(\mathbf{h}_{R1}^*)$ (one condition in (7) for $p = 1$), we see that there are $2K-1$ free parameters, which are reduced to 2 in order to satisfy the other $K-2$ conditions in (7) for $p = 5, 7, \cdots 2K-1$, and the $K-1$ conditions in (8). That is, $(2K-1)-(K-2)-(K-1) = 2$. Thus, let a, b be two scalars, let $\mathbf{A}, \mathbf{B}$ be $1 \times 2K$ vectors such that the matrix below is invertible, then the following choice of beam forming vector (for example) will satisfy all conditions:

$$\mathbf{u}_{34} = \begin{bmatrix} \mathbf{h}_{R1}^* \\ \mathbf{h}_{R2}^* \\ \mathbf{h}_{R5}^* \\ \mathbf{h}_{R6}^* \\ \vdots \\ \mathbf{h}_{R,2K}^* \\ \mathbf{A} \\ \mathbf{B} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ -h_{32} \\ 0 \\ -h_{36} \\ \vdots \\ -h_{3,2K} \\ a \\ b \end{bmatrix}. \tag{9}$$

Note that all the beam forming vectors must also be chosen to satisfy the relay power constraint P_R, but that we have sufficient degrees of freedom (choices of a,b) to ensure this, and that this will not affect the DoF in either case, as we will let $P_R \to \infty$, essentially removing the power constraint.

Still at receiver 1, once the interference signals have been neutralized and the undesired signals have been nulled (by the above choice of beam forming vectors), and the self-interference (SI) signal s_{12} has been subtracted off, the received signal in (4) becomes

$$Y_1 - SI = h_{21} s_{21} + \mathbf{h}_{R1}^* \mathbf{u}_{21} s_{21}, \tag{10}$$

from which the desired signal s_{21} can be decoded as long as $h_{21} \neq -\mathbf{h}_{R1}^* \mathbf{u}_{21}$, which we may guarantee by proper scaling of $\mathbf{u}_{21}$. Similar decoding is performed at other receivers. ■

Remark 1: To achieve $2K$ DoF we have assumed full duplex operation. If instead all nodes operate in half-duplex mode, intuitively the DoF will be halved, i.e. K. Indeed, it is trivial to achieve K DoF in a half-duplex setup: In the first time slot, all $2K$ users transmit a message, and the $2K$-antenna relay listens and decodes all $2K$ messages using a zero-forcing decoder. At time slot 2, the relay broadcasts a signal and all users listen. By careful choice of beamforming vectors as in (9), for example, each receiver receives only their desired message in this time slot. Therefore $2K$ desired messages are obtained in 2 time slots, i.e. K DoF is achievable. Note however that in the half-duplex setting, the relay is causal rather than non-causal or instantaneous.

Remark 2: The DoF of the K-pair-user two-way interference channel is known to be K [1]; Theorem 1 implies that the addition of an instantaneous $2K$-antenna relay can increase the DoF of the K-pair-user two-way IC to $2K$ – it essentially cancels out all interference in both directions simultaneously. This may have interesting design implications for full duplex two-way interference networks – i.e. the addition of a full-duplex, instantaneous MIMO relay with $2K$ antennas (for example, a pico-cell) would double the DoF to $2K$. Note that this DoF increase is due to the non-causal relaying (and interference cancelation) rather than interaction between users.

Remark 3: To achieve the maximum $2K$ DoF we need $2K$ antennas at the relay. For comments on how many DoF are achievable with a reduced number of antennas, please see [1]. We note however that we are not able to determine the minimum number of antennas needed to achieve $2K$ DoF.

IV. DoF of K-pair-user two-way interference channel with a causal MIMO relay

It is known that for one-way channels where nodes are either sources or destinations of messages but not both as in a two-way setting, the usage of feedback, causal relays (possibly with multiple antennas), and cooperation does not increase the DoF of the network [18]. Here we show that, in sharp contrast to the results in the previous section, if the relay is actually causal, it does *not* increase the DoF of the K-pair-user two-way IC beyond that of a network without the relay present, which would have K DoF ($K/2$ in each direction). Intuitively this is because a causal relay cannot mitigate the current interference signals.

We thus consider a K-pair-user two-way IC with one causal MIMO relay which has M antennas. The system model is the same as that in Section II, and the main result of this section is the following.

Theorem 2: The DoF of the K-pair-user full-duplex two-way interference channel with a causal MIMO relay is K.

Proof: Achievability follows from the fact that the DoF of the K-pair-user two-way interference channel without a relay is K, as shown in [1].

Now we prove the converse. Inspired by [18], we first transform our $2K+1$ node network to a $2K$-node network as shown in Fig. 2. Since cooperation between nodes cannot reduce the DoF, we let the causal MIMO relay fully cooperate with one of the users, take user $2K-1$ WLOG. In other words, we co-locate user $2K-1$ and the relay or put infinite capacity links between these nodes. Then the capacity region of the original network is outer bounded by that of the following $2K$-node network which each have one message and desire 1 message as before: all users except user $2K-1$ each have a single antenna, while user $2K-1$ has $M+1$ antennas (one from the original node $2K-1$, and M from the relay). Since the original relay is connected to all $2K$ users, user $2K-1$

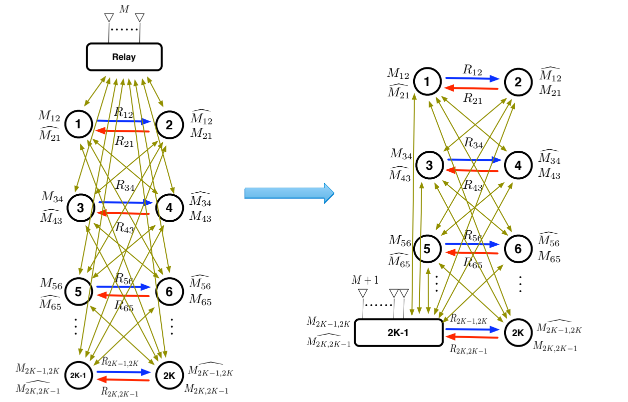

Fig. 2. Transformation of the K-pair-user full-duplex two-way interference channel with a causal MIMO relay.

in the transformed network is connected to all other users, in contrast to the original network where there is no direct link between users 1,3,...,$2K-3$ and $2K-1$. Then, letting the tilde $\tilde{A}$ notation denote the inputs, outputs and channel gains of the new network, we have the correspondences (or equivalences $\equiv$ for inputs, since they may actually be different due to interaction based on different received signals)

$$\begin{aligned}
&\tilde{X}_i \equiv X_i, \quad i = 1,2,...,2K, \text{except } 2K-1, \quad \tilde{\mathbf{X}}_{2K-1}^T \equiv [X_{2K-1}, \mathbf{X}_R^T],\\
&\tilde{Z}_i \equiv Z_i, \quad i = 1,2,...,2K, \text{except } 2K-1, \quad \tilde{\mathbf{Z}}_{2K-1}^T \equiv [Z_{2K-1}, \mathbf{Z}_R^T],\\
&\tilde{h}_{ij} = h_{ij}, \quad \text{for appropriate } i,j \text{ and } i,j \neq 2K-1\\
&\tilde{\mathbf{h}}_{i,2K-1}^T = [0, \mathbf{h}_{iR}^T], \quad i = 1,3,...,2K-3,\\
&\tilde{\mathbf{h}}_{i,2K-1}^T = [h_{i,2K-1}, \mathbf{h}_{iR}^T], \quad i = 2,4,...,2K,\\
&\tilde{\mathbf{h}}_{2K-1,j}^T = [0, \mathbf{h}_{Rj}^T], \quad j = 1,3,...,2K-3,\\
&\tilde{\mathbf{h}}_{2K-1,j}^T = [h_{2K-1,j}, \mathbf{h}_{Rj}^T], \quad j = 2,4,...,2K,
\end{aligned}$$

and the following input/output relationships at each channel use:

$$\tilde{Y}_p[k] = \sum_{m=1}^{K} \tilde{h}_{2m,p}[k]\tilde{X}_{2m}[k] + \tilde{\mathbf{h}}_{2K-1,p}^*[k]\tilde{\mathbf{X}}_{2K-1}[k] + \tilde{Z}_p[k], \quad p = 1,3,...,2K-3 \tag{11}$$

$$\tilde{Y}_q[k] = \sum_{m=1}^{K-1} \tilde{h}_{2m-1,q}[k]\tilde{X}_{2m-1}[k] + \tilde{\mathbf{h}}_{2K-1,q}^*[k]\tilde{\mathbf{X}}_{2K-1}[k] + \tilde{Z}_q[k], \quad q = 2,4,...,2K \tag{12}$$

$$\tilde{\mathbf{Y}}_{2K-1}[k] = \sum_{m=1, m\neq 2K-1}^{2K} \tilde{\mathbf{h}}_{m,2K-1}[k]\tilde{X}_m[k] + \tilde{\mathbf{Z}}_{2K-1}[k]. \tag{13}$$

We have the interactive encoding functions at each node

$$\tilde{X}_i[k] = \tilde{f}_i(M_{ij}, \tilde{Y}_i^{k-1}), \qquad i = 1,2,...,2K, \text{except } 2K-1 \tag{14}$$

$$\tilde{\mathbf{X}}_{2K-1}[k] = \tilde{f}_{2K-1}(M_{2K-1,2K}, \tilde{\mathbf{Y}}_{2K-1}^{k-1}) \tag{15}$$

where (15) is where the causality of the relay is observed / incorporated.

Let M_A denote all the messages except M_{12}, M_{34}, and let $\tilde{Y}_{(2,...,2K)/4}$ denote $\tilde{Y}_2, \tilde{Y}_3, \tilde{Y}_5, ..., \tilde{Y}_{2K}$ i.e. all outputs except $\tilde{Y}_1$ and $\tilde{Y}_4$. Note $\tilde{Y}_{(2,...,2K)/4}$ includes the output vector $\tilde{\mathbf{Y}}_{2K-1}$

at user $2K-1$. Similarly, $\tilde{X}_{(2,\ldots,2K)/4}$ and $\tilde{Z}_{(2,\ldots,2K)/4}$ denote all inputs and noises except those at nodes 1 and 4.

We now bound the sum-rate in each direction, considering the sum of a pair of rates, we will have

$$
\begin{aligned}
&n(R_{12}+R_{34}-\epsilon)\\
&\overset{(a)}{\leq} I(M_{34};\tilde{Y}_4^n|M_A)+I(M_{12};\tilde{Y}_4^n,\tilde{Y}_{(2,\ldots,2K)/4}^n|M_{34},M_A)\\
&= H(\tilde{Y}_4^n|M_A)-H(\tilde{Y}_4^n|M_{34},M_A)+H(\tilde{Y}_4^n,\tilde{Y}_{(2,\ldots,2K)/4}^n|M_{34},M_A)\\
&- H(\tilde{Y}_4^n,\tilde{Y}_{(2,\ldots,2K)/4}^n|M_{34},M_A,M_{12})\\
&\overset{(b)}{\leq} H(\tilde{Y}_4^n|M_A)+H(\tilde{Y}_{(2,\ldots,2K)/4}^n|\tilde{Y}_4^n,M_{34},M_A)-H(\tilde{Z}_4^n,\tilde{Z}_{(2,\ldots,2K)/4}^n)\\
&= H(\tilde{Y}_4^n|M_A)-H(\tilde{Z}_4^n)+H(\tilde{Y}_{(2,\ldots,2K)/4}^n|M_{34},M_A,\tilde{Y}_4^n)\\
&\quad - H(\tilde{Z}_{(2,\ldots,2K)/4}^n)\\
&\leq \sum_{k=1}^{n}[H(\tilde{Y}_4[k])-H(\tilde{Z}_4[k])+H(\tilde{Y}_{(2,\ldots,2K)/4}[k]|\tilde{Y}_{(2,\ldots,2K)/4}^{k-1},M_{34},M_A,\\
&\tilde{Y}_4^n,\tilde{X}_4^n,\tilde{X}_{(2,\ldots,2K)/4}^k)-H(\tilde{Z}_{(2,\ldots,2K)/4}[k])]\\
&\leq n(\log P+o(\log P))\\
&+\sum_{k=1}^{n}[H(\tilde{h}_{12}[k]\tilde{X}_1[k]+\tilde{Z}_2[k],\tilde{Z}_3[k],\cdots,\tilde{\mathbf{h}}_{1,2K-1}[k]\tilde{X}_1[k]\\
&\quad +\tilde{\mathbf{Z}}_{2K-1}[k],\tilde{h}_{1,2K}[k]\tilde{X}_1[k]+\tilde{Z}_{2K}[k]|\tilde{h}_{14}\tilde{X}_1[k]+\tilde{Z}_4[k])\\
&\quad - H(\tilde{Z}_{(2,\ldots,2K)/4}[k])]\\
&\overset{(c)}{\leq} n(\log P+o(\log P))+no(\log P),
\end{aligned}
$$

where (a) follows from the Fano's inequality and providing side-information to receiver 2; (b) uses conditioning reduces entropy; and (c) follows as it may be shown that the Gaussian distribution maximizes conditional entropy, as done in [18, Equation (30), (31)], similar to [19, Lemma 1]. Note also that the conditional entropy term involves a single-input, multiple output term, and hence is again bounded by $n\,o(\log P)$, due to the conditioning.

Similarly, we can derive the bound $R_{21}+R_{43}$ for the opposite direction (details please refer to [1]). Then,

$$
\begin{aligned}
d_{12}+d_{34}+d_{21}+d_{43} &\leq \limsup_{P\to\infty}\frac{R_{12}+R_{34}+R_{21}+R_{43}}{\log(P)}\\
&\leq 1+0+0+1+0+0=2,
\end{aligned}
$$

Summing over all rate pairs (see the following Remark) leads to the theorem, which indicates that the causal MIMO relay cannot increase the DoF of the full-duplex K-pair-user TWIC. ■

Remark 4: We are able to sum over all rate pairs because the asymmetry of the transformed network (multiple antennas at user $2K-1$ only, and user $2K-1$ is connected to all other nodes, unlike the even and odd numbered nodes) does not affect the DoF. Intuitively this is because for a SIMO or MISO point-to-point channel, the DoF is still 1. More rigorously, please see [1].

V. Conclusion

We proposed and studied the K-pair-user two-way interference channel with a MIMO relay where all nodes operate in full-duplex. We showed that if we introduce a $2K$ antenna, full-duplex and non-causal relay, that the DoF may be doubled over the full-duplex, relay-free counterpart (or quadrupled over the half-duplex counterpart). We demonstrated a one-shot scheme to achieve the maximal $2K$ DoF. In sharp contrast, if the relay is causal rather than non-causal, we derived a new converse showing that the DoF cannot be increased beyond K for a K-pair-user two-way full-duplex IC.

Acknowledgement

The authors would like to thank Tang Liu, a Ph.D. student at UIC, for suggesting the "one-shot" scheme in Section III, which is simpler than the authors' previously derived block Markov coding scheme, to achieve $2K$ DoF.

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