Characterization of the Effect of Radar Interference on an Uncoded Data Communication System

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***Abstract*—We investigate the effect of radar interference on an uncoded data communication system where the optimal Maximum-A-Posteriori decoder is used and the bandwidth of the radar system is assumed to be much larger than the one of the communication system. Conclusions depend on how the radar interference power, measured by the Interference-to-Noise ratio (INR), compares with the intended signal power, measured by the Signal-to-Noise ratio (SNR).**

I. Introduction

Spectrum sharing between radar and communications systems is considered because of the high demand for new wireless services and limited available bandwidth [1], [2]. Contrary to the design of communications or radar systems that operate in separate spectral bandwidths, the effective design of coexistent systems requires first understanding how current unaltered radar and communication systems would affect one another [3], [4].

There are two important aspects to examine: how communications signals affect the performance of radar systems [5], [6], [7], [8], [9], [10], [11] and viceversa, how radar signals affect the bit error rate (and other metrics) of communications signals. We are interested in analytically describing the latter.

Here, we consider the effect of radar interference on an uncoded digital data communication system where the optimal Maximum-A-Posteriori (MAP) decoder is used and where the bandwidth of the radar system is assumed to be much larger than the one of the communication system.

Conclusions on how much the radar system impacts the communication system depend on how the radar interference power, measured by the Interference-to-Noise ratio (INR), compares with the intended signal power, measured by the Signal-to-Noise ratio (SNR). Our analysis identifies three regimes of operation. We also consider two suboptimal decoders that approximate the MAP decoder at INR≪SNR and INR≫SNR.

II. System Model, Optimal MAP Decoder and Error Rate Analysis

At the communication receiver, the effect of a short duty-cycle radar pulse on a narrowband data communication system is modeled as the discrete-time complex-valued received signal

$$Y = \sqrt{\mathsf{S}}X + \sqrt{\mathsf{I}}e^{j\Theta} + Z, \tag{1}$$

where X is the transmitted symbol from the constellation $\mathcal{X} = \{x_1, ..., x_N\}$ of unit energy and equally likely points, Θ is the random phase of the radar interference uniformly distributed in $[0, 2\pi]$, and Z is a zero-mean unit-variance proper-complex Gaussian noise. The random variables (X, Θ, Z) are independent. For the (without loss of generality) normalizations used in this paper, S is the average SNR at the communication receiver, while I is the average INR. In the following we assume that the pair (S, I) is known at the receiver.

The goal is to evaluate the average probability of error, $\Pr[X \neq \hat{X}]$ where $\hat{X}$ is the estimate at the communication receiver of the transmit signal X, for the AWGN with additive radar interference in (1).

Let the channel conditional distribution be indicated as

$$f_{Y|X,\Theta} := \frac{1}{\pi} e^{-|Y - \sqrt{\mathsf{S}}X - \sqrt{\mathsf{I}}e^{j\Theta}|^2}. \tag{2}$$

The optimal MAP receiver, when the received signal is $Y = y$, chooses as estimate of the transmit constellation point

$$\hat{\ell}(y) = \arg\max_{\ell \in [1:N]} \Pr[X = x_\ell | Y = y] \tag{3}$$

$$= \arg\min_{\ell \in [1:N]} \left(|y - \sqrt{\mathsf{S}}x_\ell|^2 - \ln I_0(2\sqrt{\mathsf{I}}|y - \sqrt{\mathsf{S}}x_\ell|) \right), \tag{4}$$

where I_0 is the modified Bessel function of the first kind of order zero. At low INR, the MAP decoder in (4) can be approximated as

$$\arg\min_{\ell \in [1:N]} |y - \sqrt{\mathsf{S}}x_\ell|^2, \tag{5}$$

referred to as a “Treat Interference a Gaussian Noise” (TIN) decoder, which is exactly optimal at $\mathsf{I} = 0$. At the other extreme, when $\mathsf{I} \to \infty$ (actually when $\mathsf{I} \gg \mathsf{S} \gg 1$) the MAP decode in (4) can be approximated as

$$\arg\min_{\ell \in [1:N]} \left(|y - \sqrt{\mathsf{S}}x_\ell| - \sqrt{\mathsf{I}} \right)^2, \tag{6}$$

by using a series expansion of the I_0 function. The decoder in (6) can be interpreted as: by neglecting the noise, then $Y \approx \sqrt{\mathsf{S}}X + \sqrt{\mathsf{I}}\ e^{j\Theta}$; for the correct symbol $|Y - \sqrt{\mathsf{S}}x_\ell| \approx |\sqrt{\mathsf{I}}\ e^{j\Theta}| = \sqrt{\mathsf{I}}$, thus $|Y - \sqrt{\mathsf{S}}x_\ell| - \sqrt{\mathsf{I}} \approx 0$, while for other symbols the decoding metric will be bigger than zero.

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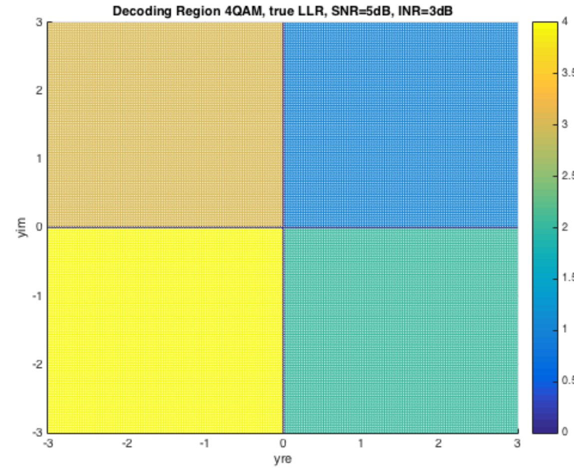
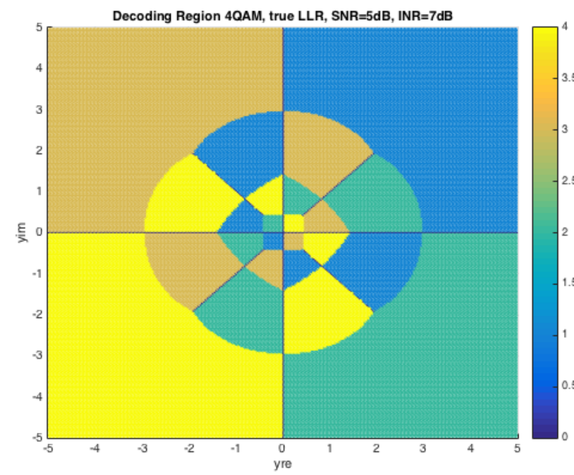
For the 2-PAM case, decoder in (6) was studied in [12]. Here we consider the 4-QAM case. Fig. 1 shows the optimal decoding regions based on the the MAP decoder in (4) for a 4-QAM. In Fig. 1a, where the INR is smaller than the SNR, the optimal decoding regions are as for the case $\mathsf{I} = 0$; in this regime the MAP decode reduces to the one in (5). In Fig. 1b, where INR is larger than the SNR, the decoding region have a very interesting 'arty-farty' shape that turns out to be well approximated by the decoder in (6).

Fig. 2 shows the Symbol Error Rate (SER) performance for various decoders when $\mathsf{S} = 5$ dB as a function of I in dB. When there is no interference, the SER is given by the lower bound curve. In the presence of interference, the SER increases as shown by the optimal decoder curve given by equation (4). When I is low compared to S, the performance of the MAP and TIN decoder coincide. When I is large compared to S, the performance of the approximate decoder in (6) agrees with that of the MAP decoder. Overall, the approximate decoders do not agree well with the MAP decoder only when $\mathsf{I} \approx \mathsf{S}$; this is also the regime where the SER attains its highest value and it is thus the regime where one would not want the two systems to coexist.



(a) INR=3 dB



(b) INR=7 dB

Fig. 1: Optimal decision regions for 4QAM signaling.

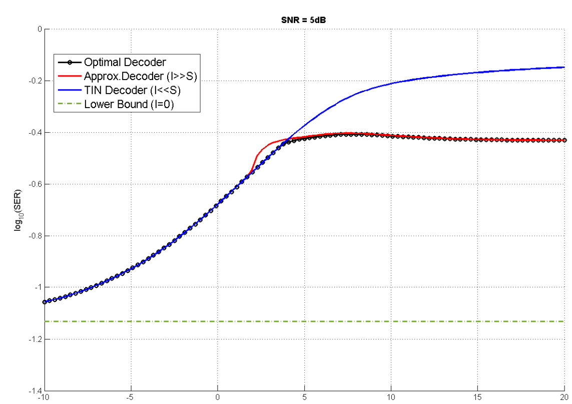


Fig. 2: BER for 4QAM with SNR=5 dB vs INR in dB.

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