

# On the Sum-Capacity of the Cognitive Interference Channel with Cognitive-Only Message Sharing

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**Abstract**—Motivated by the ongoing discussion of spectrum scarcity, this paper considers the  $K$ -user cognitive interference channel with  $K - 1$  primary/licensed users and one cognitive/secondary user who has non-causal knowledge of the messages of all primary users. This message sharing mechanism is referred to as *cognitive-only message sharing*. For certain parameter regimes, the sum-capacity of the symmetric Gaussian noise channel is characterized to within an additive constant gap from an outer bound originally derived for a channel model with *cumulative message sharing*, which consists of one primary user and  $K - 1$  cognitive users where cognitive transmitter  $i \in [2 : K]$  has non-causal knowledge of the messages of the users with index less than  $i$ . The approximately optimal achievability scheme is a combination of simultaneous interference neutralization at the primary receivers, dirty-paper coding to remove the effect of interference at the cognitive receiver, and rate-splitting, where the power splits are chosen such that the signals treated as noise are received below the noise floor of the receiver. This shows that “distributed cognition” may not be necessary in the considered network model since (approximately) the same sum-capacity can be achieved by having only one “globally cognitive” user whose role is to manage all the interference in the network.

## I. INTRODUCTION

Cognitive radio (CR) is one of the most promising technologies to revolutionize current spectrum management [1]. Cognitive devices are advanced software defined radios that are aware of their surrounding environment and can exploit the underutilized spectrum. According to the type of side information about the environment, CRs are classified in the following three types [1]: (1) *interweave*: a CR can opportunistically communicate over white spaces (time, space or frequency void) by detecting the primary users’ transmission in the network, or (2) *underlay*: a CR can communicate simultaneously with primary users if the interference caused at the primary receivers is kept below a certain threshold commonly referred to as the interference temperature, or (3) *overlay*: a CR can use sophisticated encoding and decoding schemes to aid the primary users’ transmission.

*Past Work*: The overlay cognitive radio channel, first introduced in [2], consists of two interfering transmitter-receiver pairs in which one transmitter, referred to as *secondary*, has non-causal knowledge of the message of the other transmitter, known as *primary*. The most comprehensive results on the two-user Cognitive Interference Channel (2-CIFC) can be found in [3]–[5]. In particular, for the practically relevant Gaussian noise channel, the capacity region is known exactly for some channel parameters and to within a number of

bits proportional to the number of antennas of the secondary receiver [5].

Several  $K$ -user extensions of the 2-CIFC model have been studied in the literature. For the 3-user case, the 3-CIFC, the work [6] proposed the following models for cognition/message sharing: that with *cumulative message sharing* (3-CIFC-CMS), that with *primary message sharing* (3-CIFC-PMS), and that with *cognitive-only message sharing* (3-CIFC-CoMS). In the 3-CIFC-CMS, there are 2 cognitive users: transmitter 2 knows the message of primary user 1, and transmitter 3 knows the messages of users 1 and 2. In the 3-CIFC-PMS, there are 2 cognitive users: transmitters 2 and 3, who only know the message of the primary user 1. In the 3-CIFC-CoMS there are two primary users who do not know each others’ message and a single cognitive user who knows both primary messages.

In [7], the 3-user PMS and CMS, while in [8] the 3-user CoMS scenario were introduced. Achievable rate regions were obtained and were numerically evaluated for the Gaussian noise channel; a full comparison was made in [6]. A special case of the 3-CIFC-CoMS in which the cognitive user is assumed not to interfere with the primary users was studied; an inner and an outer bound are obtained in [9], while capacity under strong interference was obtained in [10], [11].

The case of more than three users is far less understood. In [12] the authors consider a channel model that consists of one primary user and  $K - 1$  cognitive users, a  $K$ -user extension of the PMS scenario: each cognitive user only knows the primary message in addition to their own message. However, they restrict the channel so that the cognitive users do not cause interference to one another but only to the primary receiver and are interfered only by the primary transmitter; for this channel model the capacity in the “very strong” interference regime is obtained by using lattice codes. In [13] the Degrees of Freedom (DoF) of a  $K$ -user interference channel in which each transmitter, in addition to its own message, has access to a subset of the other users’ messages, was obtained; in particular, it was shown that the  $\text{DoF} = K$  if the sum of the number of jointly cooperating transmitters and the jointly decoding receivers is greater than or equal to  $K + 1$ . In [14] the sum-capacity of a  $K$ -CIFC-CoMS and a  $K$ -CIFC-CMS under certain “strong interference” conditions was derived; it was shown that simply beam-forming to the primary receiver was sum-capacity achieving.

In [15] we characterized the sum-capacity of the symmetric linear deterministic  $K$ -CIFC-CMS; since the achievable

scheme required only cognitive message knowledge at one user, i.e., corresponding to CIFIC-CoMS, we suspected that the sum-capacity upper bound for the symmetric Gaussian  $K$ -CIFIC-CMS could be achieved, to within a constant gap, with the much less demanding message structure of the  $K$ -CIFIC-CoMS. In [15] we derived the sum-capacity for the symmetric Gaussian  $K$ -CIFIC-CMS by an achievability scheme inspired by the MIMO broadcast channel; although the message knowledge needed at the transmitters was much simplified compared to the CMS assumption, it did not correspond to the CoMS. *In this paper we show that indeed the sum-capacity upper bound for the symmetric Gaussian  $K$ -CIFIC-CMS is to within a constant gap (in some regimes) of an achievable scheme designed for the symmetric Gaussian  $K$ -CIFIC-CoMS.* We notice that the  $K$ -CIFIC-CoMS model reduces to that of a  $(K - 1)$ -IFC with a *cognitive relay* if the secondary user does not send its own message. We shall leverage the results of [16]–[18] when studying the sum-capacity of the 3-CIFIC-CoMS.

*Contributions and Paper Organization:* In this paper we consider the  $K$ -CIFIC-CoMS for arbitrary  $K$ . For the symmetric Gaussian noise channel, we show that the sum-capacity outer bound initially derived in [15] for the  $K$ -CIFIC-CMS is achievable to within a constant gap by a scheme that requires only the message structure of the  $K$ -CIFIC-CoMS. This result is shown to hold for certain channel gain relationships: in one regime it is either optimal for the cognitive user to behave as a cognitive relay as the direct link of the cognitive user is so weak, while in a second regime the cognitive user’s rate is strong enough to warrant non-zero rate. In both regimes we impose that the channel gains from the cognitive transmitter to the primary receivers are at least twice (for the 3-user case) as strong as the interfering channel gains, as our achievability schemes require that the cognitive transmitter is able to cancel the interference at both primary receivers simultaneously. This idea is then extended to any  $K$ . Our result shows that “distributed cognition” in the  $K$ -CIFIC-CMS model may not be necessary in the considered network since (approximately) the same sum-capacity can be achieved by having only one “globally cognitive” user whose role is to manage all the interference in the network.

The channel model is described in Section II. We report the detailed proofs for the case of  $K = 3$  users in Section III-B and discuss the generalization to any  $K$  in Section III-C. For the  $K$ -user case the details are omitted for sake of space. Section IV concludes the paper.

## II. CHANNEL MODEL

The  $K$ -CIFIC-CoMS consists of  $K - 1$  primary (each have an independent message) and one cognitive (with knowledge of all messages) transmitters and  $K$  receivers (each interested in one message only). The Gaussian channel is described by the following input-output relationship

$$Y_j = \sum_{k \in [1:K]} h_{jk} X_k + Z_j, \quad j \in [1 : K], \quad (3)$$

with the usual assumptions: the complex-valued channel gains are constant and known to all terminals, the inputs are subject

to power constraints  $\mathbb{E}[|X_i|^2] \leq 1$ ,  $i \in [1 : K]$ , the outputs are subject to Gaussian noise  $Z_i \sim \mathcal{N}(0, 1)$ ,  $i \in [1 : K]$ . Our notation, and definitions of achievable rates, capacity region and sum-capacity follow [19].

In the following we shall focus on the *symmetric* case defined by: for  $j \in [1 : K - 1]$ ,

$$h_{jj} = |h_d|, \quad (\text{primary direct links}), \quad (4a)$$

$$h_{jK} = h_c, \quad (\text{secondary} \rightarrow \text{primary links}), \quad (4b)$$

$$h_{jk} = h_i, \quad k \notin \{j, K\} \quad (\text{primary interfering links}). \quad (4c)$$

Note that our “symmetric setting” in (4) makes the primary users completely equivalent but does not impose any restriction on the channel gains of the cognitive receiver (i.e.,  $h_{Ki}$ ,  $i \in [1 : K]$ ) which are kept general.

The 3-CIFIC-CoMS is shown in Fig. 1(a) and the 3-CIFIC-CMS in Fig. 1(b). Notice the different message structure at the transmitters, clearly the capacity of the 3-CIFIC-CMS is an outer bound to the capacity of the 3-CIFIC-CoMS. For the 3-user case, the condition in (4) means that the two primary direct links are equal and are denoted by  $|h_d|$  (they can be taken to be real-valued without loss of generality because a receiver can compensate for the phase of one of its channel gains), the interference cross gains are equal and denoted by  $h_i$ , and the “cooperative” cross channel gains from the cognitive transmitter are equal and denoted by  $h_c$ .

## III. MAIN RESULTS

### A. Outer bound

In [15] we considered the general  $K$ -CIFIC-CMS in which transmitter  $i \in [1 : K]$  has non-causal message knowledge of the messages of the users with index in  $[1 : i]$ . We proved that the capacity region of the general memoryless  $K$ -CIFIC-CMS is contained into the region [15, Theorem 2]: for  $i \in [1 : K]$

$$R_i \leq I(Y_i; X_{[i:K]} | X_{[1:i-1]}), \quad (5a)$$

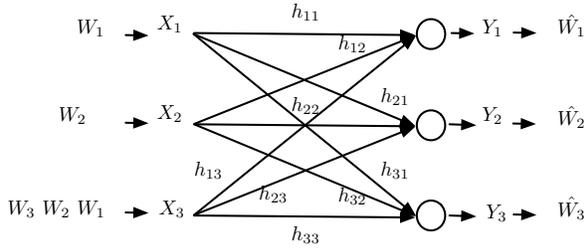
$$\sum_{j=i}^K R_j \leq \sum_{j=i}^K I(Y_j; X_{[j:K]} | X_{[1:j-1]}, Y_{[1:j-1]}), \quad (5b)$$

for some joint input distribution  $P_{X_1, \dots, X_K}$ , where  $X_S := \{X_i : i \in S\}$  for some index set  $S \subseteq [1 : K]$ .

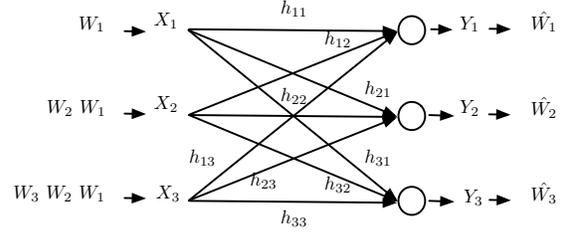
An outer bound for the  $K$ -CIFIC-CoMS, where transmitter  $i \in [1 : K - 1]$  only knows its own message, can be obtained by giving message side information to the  $K - 1$  primary users so as to transform the CoMS message structure (see Fig. 1(a)) into the CMS one (see Fig. 1(b)). For each possible permutation of the primary users’ indices we obtain a region as in (5); by intersection of all these  $(K - 1)!$  outer bound regions we obtain an outer bound for the channel of interest; by Fourier-Motzkin elimination a sum-rate upper bound can be obtained. We illustrate the achievability of the symmetric version of this sum-capacity upper bound by detailing the case  $K = 3$  first and then generalize it to any  $K$ .

### B. $K = 3$ user case

For the 3-user case, the outer bound in Section III-A gives the sum-rate in (1) at the top of the page. Note that



(a)  $K$ -CIFC-CoMS with  $K = 3$ .



(b)  $K$ -CIFC-CMS with  $K = 3$ .

$$R_1 + R_2 + R_3 \leq I(Y_1; X_1, X_3 | X_2) + I(Y_2; X_2, X_3 | X_1) + \min \left\{ I(Y_3; X_3 | X_1, X_2, Y_1), I(Y_3; X_3 | X_1, X_2, Y_2) \right\}, \quad (1a)$$

$$R_1 + R_2 + R_3 \leq \min \left\{ I(Y_1; X_1, X_2, X_3) + I(Y_2; X_2, X_3 | X_1, Y_1), I(Y_1; X_1, X_3 | X_2, Y_2) + I(Y_2; X_1, X_2, X_3) \right\} + I(Y_3; X_3 | X_2, Y_2, X_1, Y_1), \quad \text{for some input distribution } P_{X_2, X_1, X_3}. \quad (1b)$$

$$R_1 + R_2 + R_3 \leq 2 \log(1 + (|h_d| + |h_c|)^2) + \log \left( 1 + \frac{|h_{33}|^2}{1 + |h_c|^2} \right), \quad (2a)$$

$$R_1 + R_2 + R_3 \leq \log(1 + (|h_d| + |h_i| + |h_c|)^2) + \log(2) + \log \left( 1 + \frac{||h_d| - h_i|^2}{2} \right) + \log \left( 1 + \frac{|h_{33}|^2}{1 + 2|h_c|^2} \right). \quad (2b)$$

although the channel model imposes an input distribution that factors as  $P_{X_2, X_1, X_3} = P_{X_1} P_{X_2} P_{X_3 | X_2, X_1}$ , our bound has to be evaluated over all possible joint input distributions since message side information was given in the converse.

*Remark 1.* The mutual information terms involving  $Y_3$ , thus characterizing the rate of the secondary user/user 3 (i.e., that depend on  $Y_3$ ) in (1) are all conditioned on the primary users' signals ( $X_1, X_2$ ). For the Gaussian channel this implies that the sum-capacity outer bound in (1) does *not* depend on the channel gains ( $h_{31}, h_{32}$ ). From an achievability perspective, this may be interpreted as follows: since transmitter 3 has a priori knowledge of the messages of users 1 and 2, by Dirty Paper Coding (DPC) it can "pre-cancel" the effect of  $X_1$  and  $X_2$  at  $Y_3$  thus effectively making  $(h_{31}, h_{32}) = (0, 0)$ . The same holds for any  $K$ : the channel gains  $h_{Kj}, j \in [1 : K - 1]$  do not affect the outer bound described in Section III-A.

For the Gaussian noise channel, the outer bound in (1) is exhausted by jointly Gaussian inputs – by the "Gaussian maximizes entropy"-principle [19] – and can be further upper bounded as in (2) at the top of the page, as shown in [15].

Our main result is as follows

**Theorem 1.** *For the 3-CIFC-CoMS, let*

$$\text{Case 1: } |h_{33}|^2 \leq |h_c|^2, \quad 2|h_i|^2 \leq |h_c|^2 \leq |h_d|^2, \quad (6)$$

$$\text{Case 2: } |h_{33}|^2 > |h_c|^2, \quad 2|h_i|^2 \leq |h_c|^2 \leq |h_d|^2. \quad (7)$$

*The sum-capacity bound in (2b) is achievable to within 8.4 bits when (6) is satisfied, and to within 13 bits when (7) is satisfied.*

*Proof:* We consider the following transmit signals

$$X_1 = \alpha_1 T_{1ZF} + \gamma_1 T_{1p}, \quad (8a)$$

$$X_2 = \beta_2 T_{2ZF} + \gamma_2 T_{2p}, \quad (8b)$$

$$X_3 = -\alpha_3 T_{1ZF} - \beta_3 T_{2ZF} + 0 \cdot T_{3ZF} + \gamma_3 T_{3p}, \quad (8c)$$

where  $T_{iZF}, T_{ip}$  (ZF stands for zero forcing, p stands for private) are independent  $\mathcal{N}(0, 1)$  random variables for  $i \in [1 : 3]$  and the coefficients are such that

$$|\alpha_1|^2 + |\gamma_1|^2 \leq 1, \quad (8d)$$

$$|\beta_2|^2 + |\gamma_2|^2 \leq 1, \quad (8e)$$

$$|\alpha_3|^2 + |\beta_3|^2 + |\gamma_3|^2 \leq 1, \quad (8f)$$

in order to satisfy the power constraints. With (8), the received signals are

$$Y_j = (h_{j1}\alpha_1 - h_{j3}\alpha_3)T_{1ZF} + h_{j1}\gamma_1 T_{1p} + (h_{j2}\beta_2 - h_{j3}\beta_3)T_{2ZF} + h_{j2}\gamma_2 T_{2p} + h_{j3}\gamma_3 T_{3p} + Z_j, \quad j \in [1 : 3].$$

When  $|h_{33}|^2 \leq |h_c|^2$ , the terms in (2) depending on  $h_{33}$ , which give the rate for the cognitive user, are bounded by  $\log \left( 1 + \frac{|h_{33}|^2}{1 + 2|h_c|^2} \right) \leq \log \left( 1 + \frac{|h_{33}|^2}{1 + |h_c|^2} \right) \leq \log(2)$ . This seems to suggest that it is optimal, to within a constant gap, to have  $R_3 = 0$  when  $|h_{33}|^2 \leq |h_c|^2$ , that is, to have the cognitive user use all its resources to help the primary users / behave as a *cognitive relay*. The symmetric capacity region of a 2-user interference channel with cognitive relay was recently characterized to within a constant gap in [18] for almost all parameter regimes.

In this paper we use a technique to upper bound (2b) that is tailored to the symmetric case. The power splits are chosen

$$\gamma_1 = \gamma_2 = \gamma_3 = 0, \quad (9a)$$

$$\alpha_1 = \beta_2 = \alpha_3 = \beta_3 = \frac{h_i}{h_c}. \quad (9b)$$

Having the interference completely neutralized, the following rates are achievable

$$R_1 \geq \log \left( 1 + |h_d - h_i|^2 \right), \quad (10a)$$

$$R_2 \geq \log \left( 1 + |h_d - h_i|^2 \right). \quad (10b)$$

Note that we can further lower bound  $R_1$  using (6) by

$$R_1 \geq \log \left( 1 + (|h_d| - |h_i|)^2 \right) \geq \log \left( 1 + \left( 1 - \frac{1}{\sqrt{2}} \right)^2 |h_d|^2 \right).$$

We next use the condition in (6) to further upper bound the first expression of the sum-capacity upper bound in (2b) as

$$\log \left( 1 + (|h_d| + |h_i| + |h_c|)^2 \right) \leq \log \left( 1 + \left( 2 + \frac{1}{\sqrt{2}} \right)^2 |h_d|^2 \right).$$

Finally, by taking the difference between upper and lower bounds we get (note that  $R_3 \leq \log_2(2)$ )

$$\begin{aligned} \text{gap} &\leq \log \left( 1 + \left( 2 + \frac{1}{\sqrt{2}} \right)^2 |h_d|^2 \right) + \log \left( 1 + \frac{|h_d - h_i|^2}{2} \right) \\ &\quad - \log \left( 1 + \left( 1 - \frac{1}{\sqrt{2}} \right)^2 |h_d|^2 \right) - \log \left( 1 + |h_d - h_i|^2 \right) \\ &\quad + 2 \log(2) \leq \log \left( 4 \frac{\left( 2 + \frac{1}{\sqrt{2}} \right)^2}{\left( 1 - \frac{1}{\sqrt{2}} \right)^2} \right) \approx 8.4 \text{ bits.} \end{aligned}$$

When  $|h_{33}|^2 > |h_c|^2$ , the outer bound in (2) suggests that the intended signal at the cognitive receiver is strong enough to support  $R_3 > 0$ . We focus on the regime identified by (7). User  $i \in [1 : 2]$  splits its message into two parts: private information (to be kept below the noise floor at the non-intended primary receiver) and zero-forced information (to be zero-forced by the cognitive transmitter at the non-intended primary receiver). The cognitive transmitter pre-codes against the whole interference seen at its receiver by using Dirty Paper Coding (DPC) so that its receiver does not experience interference from the primary users.

Next we choose the power splits so as to match the upper bound. In order to zero-force / neutralize the interference of a primary user at the non-intended primary receiver we set  $h_{21}\alpha_1 = h_{23}\alpha_3$  and  $h_{12}\beta_2 = h_{13}\beta_3$ . Moreover, since the interfering  $T_{jp}$ 's are considered as noise at the primary receivers we set  $|h_{12}\gamma_2| \leq 1$ ,  $|h_{13}\gamma_3| \leq 1$ ,  $|h_{21}\gamma_1| \leq 1$ ,  $|h_{23}\gamma_3| \leq 1$ . For the symmetric channel, under the condition in (7) this can be accomplished by setting

$$|\gamma_1|^2 = 1 - |\alpha_1|^2 = |\gamma_2|^2 = 1 - |\beta_2|^2 = |\gamma_3|^2 = \frac{1}{1 + 2|h_c|^2}, \quad (11a)$$

$$\alpha_3 = \beta_3 = \frac{h_i}{h_c} \sqrt{1 - \frac{1}{1 + 2|h_c|^2}}. \quad (11b)$$

With (11) and under the condition in (7), the variance of the overall noise (i.e., actual noise plus the interfering signals treated as noise) is upper bounded as

$$1 + \frac{|h_c|^2 + |h_i|^2}{1 + 2|h_c|^2} \leq 1 + \frac{(1 + 1/2)|h_c|^2}{1 + 2|h_c|^2} \leq 1.75. \quad (12)$$

With (12), we see that the rate of the primary users satisfy

$$\begin{aligned} R_1 &= R_2 \\ &\geq \log \left( 1 + \frac{\left( 1 - \frac{1}{1 + 2|h_c|^2} \right) |h_d - h_i|^2 + \frac{|h_d|^2}{1 + 2|h_c|^2}}{1.75} \right) \\ &\geq \log \left( 1 + \frac{\left( 1 - \frac{1}{\sqrt{2}} \right)^2}{1.75} |h_d|^2 \right), \end{aligned} \quad (13)$$

while for the cognitive user, who DPCs against the whole interference due to the primary users, we have

$$R_3 = \log \left( 1 + \frac{|h_{33}|^2}{2|h_c|^2 + 1} \right). \quad (14)$$

We next use the condition in (7) to further upper bound the first two expressions of the sum-capacity outer bound in (2b) as

$$\begin{aligned} \log \left( 1 + (|h_d| + |h_i| + |h_c|)^2 \right) &\leq \log \left( 1 + \left( 2 + \frac{1}{\sqrt{2}} \right)^2 |h_d|^2 \right), \\ \log \left( 1 + |h_d - h_i|^2 \right) &\leq \log \left( \left( 1 + \frac{1}{\sqrt{2}} \right)^2 \frac{|h_d|^2}{2} \right) \end{aligned}$$

Finally, by taking the difference between upper and lower bounds we get (note that the terms depending on  $|h_{33}|^2$  match exactly)

$$\begin{aligned} \text{gap} &\leq \log \left( 1 + \left( 2 + \frac{1}{\sqrt{2}} \right)^2 |h_d|^2 \right) + \log \left( \left( 1 + \frac{1}{\sqrt{2}} \right)^2 \frac{|h_d|^2}{2} \right) \\ &\quad - 2 \log \left( 1 + \frac{\left( 1 - \frac{1}{\sqrt{2}} \right)^2}{1.75} |h_d|^2 \right) + \log(2) \\ &\leq \log(2) + \log \left( \frac{\left( 2 + \frac{1}{\sqrt{2}} \right)^2}{\left( \frac{1 - \frac{1}{\sqrt{2}}}{1.75} \right)^2} \right) + \log \left( \frac{\left( 1 + \frac{1}{\sqrt{2}} \right)^2}{2 \left( \frac{1 - \frac{1}{\sqrt{2}}}{1.75} \right)^2} \right) \\ &\approx 13 \text{ bits.} \end{aligned}$$

This concludes the proof.  $\blacksquare$

### C. Extension to the $K$ -user case

Based on the intuition developed in the previous section, we now describe a scheme for the symmetric  $K$ -CIFC-CoMS for any  $K$  that achieves to within a constant gap the outer bound of the  $K$ -CIFC-CMS. In [15] we showed that the sum-capacity of the symmetric Gaussian  $K$ -CIFC-CMS is upper bounded by

$$\begin{aligned} \sum_{k=1}^K R_k &\leq \log \left( 1 + (|h_d| + (K - 2)|h_i| + |h_c|)^2 \right) \\ &\quad + (K - 2) \log \left( 1 + \frac{|h_d - h_i|^2}{2} \right) + (K - 2) \log(2) \\ &\quad + \log \left( 1 + \frac{|h_{33}|^2}{1 + (K - 1)|h_c|^2} \right). \end{aligned} \quad (15)$$

Similarly to Theorem 1 we consider the different cases.

**Theorem 2.** For the following channel gain relationships

$$|h_{33}| \leq |h_c|^2, (K-1)|h_i|^2 \leq |h_c|^2 \leq |h_d|^2, \quad (16)$$

$$|h_{33}| > |h_c|^2, (K-1)|h_i|^2 \leq |h_c|^2 \leq |h_d|^2. \quad (17)$$

The sum-capacity in (15) is achievable to within

$$\text{gap} \leq \log_2 \left( \frac{(2\sqrt{K-1} + K - 2)^2}{(\sqrt{K-1} - 1)^2} \right) + (K-1) \log_2(2)$$

bits when (16) is satisfied, and to within

$$\text{gap} \leq \log_2 \left( \left( \frac{K^2 - 2}{(K-1)^2} \right) \frac{(2\sqrt{K-1} + K - 2)^2}{(\sqrt{K-1} - 1)^2} \right) + (K-2) \log_2 \left( \frac{(K^2 - 2)(\sqrt{K-1} + 1)^2}{(K-1)^2 (\sqrt{K-1} - 1)^2} \right)$$

bits when (17) is satisfied.

*Proof:* The transmit signals are

$$X_i = \alpha_i T_{iZF} + \gamma_i T_{ip}, i \in [1 : K-1], \quad (18a)$$

$$X_K = -\beta_K \sum_{i=1}^{K-1} T_{iZF} + \gamma_K T_{Kp}. \quad (18b)$$

where  $T_{iZF}, T_{ip}$ , i.i.d  $\mathcal{N}(0, 1)$ ,  $i \in [1 : K]$  and the following constraints should be satisfied

$$|\alpha_i|^2 + |\gamma_i|^2 \leq 1, i \in [1 : K-1] \quad (19a)$$

$$|\gamma_K|^2 + (K-1)|\beta_K|^2 \leq 1. \quad (19b)$$

Consider the case defined in (17), then (12) can be generalized to  $1 + \frac{1}{K-1} + \frac{K-2}{(K-1)^2} = \frac{K^2-2}{(K-1)^2}$  for the  $K$  user case. The following rates are achievable  $R_i \geq \log \left( 1 + \frac{(K-1)^2 (\sqrt{K-1}-1)^2 |h_d|^2}{K^2-2} \right)$ ,  $i \in [1 : K-1]$  and  $R_K \leq \log \left( 1 + \frac{|h_{33}|^2}{1+(K-1)|h_c|^2} \right)$ . The first and second term in (15) can be upper bounded by  $\log \left( 1 + |h_d|^2 \frac{(2\sqrt{K-1}+K-2)^2}{\sqrt{K-1}} \right)$  and  $(K-2) \log \left( 1 + \frac{|h_d|^2}{2} \left( 1 + \frac{1}{\sqrt{K-1}} \right)^2 \right)$  respectively. The rest of the proof which includes computation of the gap and the choice of the power splits are omitted for the sake of space. We note that the gap is linear as a function of  $K$ . ■

#### IV. CONCLUSION

We have shown that the sum-capacity upper bound of a  $K$ -CIFC-CMS can be achieved with a scheme that requires only one cognitive user  $K$ -CIFC-CoMS. This indicates that “distributed cognition” or having a cumulative message knowledge structure at nodes may not be worth the overhead as (approximately) the same sum-capacity can be achieved by having only one “globally cognitive” user whose role is to manage all the interference in the network.

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