Signal Constellation Design in the Presence of Radar Interference and Gaussian Noise

Narueporn Nartasilpa, Daniela Tuninetti, Natasha Devroye
University of Illinois at Chicago, Chicago, IL 60607, USA
Email: {nnarta2,danielat,devroye}@uic.edu

***Abstract*—Increased demand for wireless services has spurred research into spectrum co-existence between radar and communication systems. The effect of an unaltered radar interference on an uncoded communication receiver has been investigated for constellations used in commercial systems. However, the constellation design – possible for a cognitive transmitter-receiver pair – that maximizes the transmission rate for communication systems affected by Gaussian noise and radar interference has not yet been addressed and is tackled here. Two regimes are investigated: when the Interference-to-Noise Ratio (INR) of the radar signal is very low and very high compared to the Signal-to-Noise Ratio (SNR) of the communication signal. It is found that: (a) When INR≪SNR, the channel behaves as a complex-valued AWGN channel; the designed constellations exhibit a concentric (almost equilateral) hexagonal structure as the number of signal points increases, (b) When INR≫SNR≫1, the channel behaves as a real-valued phase-fading AWGN channel; the designed constellations are shaped as an (almost equally-spaced) pulse-amplitude modulation as the number of signal points increases, and (c) The designed constellations outperform practically-used ones in terms of the transmission rate.**

I. Introduction

This paper considers a transmitter wishing to communicate with a receiver over a memoryless additive noise channel, where the noise consists of a Gaussian component and a radar interference. The radar interference is modeled as having a constant amplitude and a random phase. Many classical complex-valued modulation schemes, such as QAM and PSK, are widely used in various communication systems; however, these modulation schemes are not necessarily optimal for this radar interfered channel. The objective of this study is to design constellations that maximize the transmission rate, measured by the number of signal constellation points, under the certain power and Symbol Error Rate (SER) constraints.

A. Past Work

Much recent work has looked at issues related to the co-existence between radar and communication systems from a radar perspective, i.e., by optimizing the radar transmitter so as to mitigate its impact on a communication receiver. The following list is, due to space limitation, a very small extract of the available literature on the topic. In [1], the authors looked at how to design the radar system so as to maximize the Signal-to-Interference-plus-Noise-Ratio (SINR) at the MIMO radar receiver subject to MIMO communication system requirements. In [2], the radar waveforms were optimized so as to maximize the radar estimation rate as well as the communications data rate. In [3], the authors discussed the gray space spectrum sharing between a primary rotating radar and a secondary cellular device that allows the secondary device to transmit as long as its resulting interference does not exceed the radar's tolerable level.

In general the focus has been mainly on the design of radar waveforms for enhanced spectral efficiency when co-existing with communication systems. In this work, we are interested in the complementary problem: design the communication signal constellation so as to optimize the communications system performance in the presence of a radar signal. Here, in contrast to other work, the radar signal is left unaltered.

Past work has studied the optimal constellation design under various performance metrics for different channel models. However, to the best of our knowledge, no one has looked at the channel model with additive radar interference considered here. For example, the optimization of complex signal constellations in the presence of Gaussian noise only was presented in [4], where the minimization of the probability of error was based on an asymptotic approximation of the Maximum-Likelihood (ML) decoder for large Signal-to-Noise-Ratio (SNR). Another example is [5] where the authors presented the optimal signal constellation for phase noise channels when using either the SER or the average mutual information for an ML detector as the optimization objectives.

In this work, motivated by the necessity of delivering the largest possible rate subject to an error rate being below an application-dependent value while not exceeding a power budget, we study the problem of maximizing the cardinality of the constellation subject to average constellation power and SER constraints. This is related to the more common approach of minimizing the SER for a fixed constellation size subject to an average power constraint. Both problems are interesting; ours resembles a more information theoretic approach in which rate (like capacity) is the objective to be maximized.

This problem is well motivated for cognitive networks in which the communication nodes are "intelligent" and may adapt their transmission – in this case the constellation used – to their sensed spectral environment. In particular, if they sense a radar signal, depending on its strength, they may adapt their constellation and decoding schemes as proposed here.

B. Contribution

In this work we aim to design signal constellations that have the largest number of points subject to the SER (with an ML detector) not exceeding a fixed value for a power-constrained

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additive noise channel, where the noise has both Gaussian and radar interference components. Our prior work [6] reported the SER performance of constellations commonly used in practice, such as PAM, QAM and PSK, for this channel model. We focus here on two regimes, based on the relative value of the Interference-to-Noise Ratio (INR) of the interfering radar signal and the SNR of the desired communication signal. Our findings are as follows.

Low INR Regime: When INR≪SNR, the optimal ML receiver reduces to the classical minimum distance decoder for complex-valued Gaussian noise channels. Our optimization results show that the designed constellation is approximately shaped as a hexagonal lattice. More precisely, the signal constellation points are the vertices of a trellis of almost equilateral triangles that form concentric almost equilateral hexagons as the number of signal points increases. We shall refer to this shape as 'hexagonal-like'. These two-dimensional hexagonal-like constellations are optimal in complex-valued Gaussian noise channels at high SNR as shown in [4, Fig. 6].

High INR Regime: When INR≫SNR, the optimal ML receiver estimates and subtracts the radar interference from the received signal; however, this also cancels a part of the communication signal resulting in an irreducible error floor [6]. Our optimization results show that the designed constellation is approximately shaped an unequally-spaced Pulse-Amplitude Modulation (PAM). More precisely, the constellation aligns all the points on an almost straight line with the largest possible minimum distances. We shall refer to this shape as 'PAM-like'. These one-dimensional PAM-like constellations outperform commonly-used ones in terms of SER in real-valued phase-fading Gaussian channels.

Intermediate INR Regime: When INR≊SNR, this is the worst operating regime as the SER attains its highest value for a fixed SNR and a fixed constellation [6, Fig. 1(b) and 1(d)]. The constellation design in the mid INR regime is quite complex and is ongoing work.

II. System Model and Performance Metric

A. Channel Model

We start by reviewing our system model for an uncoded communication receiver in the presence of radar signal interference [6]. The radar system of interest transmits a periodic pulse train with short duty-cycle, i.e., a wideband signal. At the communication receiver of a narrowband data communication system, the effect of such a radar signal is modeled as an additive term having deterministic amplitude and uniformly distributed random phase. It has been shown in [7] that the joint Probability Mass Function (PMF) of the radar amplitude and phase consists of a union of multiple constant amplitude, uniform phase pieces. In other words, for several amplitudes, the joint PMF is uniformly distributed across the phase (for a given amplitude). One of the amplitudes clearly dominates the others, and hence the radar signal's joint PMF can be approximated by a phase uniform in $[0, 2\pi]$ at the given dominant amplitude. The discrete-time complex-valued received signal is given by

$$Y = \sqrt{\mathsf{S}}X + \sqrt{\mathsf{I}}e^{j\Theta} + Z, \tag{1}$$

where: X is the equally-likely unit-energy transmitted symbol from the complex-valued signal constellation, Θ is the random phase of the radar interference that is uniformly distributed in $[0, 2\pi]$, and Z is the zero-mean unit-variance proper-complex Gaussian noise. The random variables (X, Θ, Z) are mutually independent. Without loss of generality, S denotes the average SNR, and I denotes the average INR. We assume that the pair (S, I) is fixed. In the following, we shall consider the SER for this channel under the assumption of uncoded transmissions and optimal (in the sense of minimizing the SER) detection.

B. Optimal Decoder

The optimal ML decision rule for the received signal $Y = y$, can be expressed as [6, eq.(3)]

$$\hat{\ell}^{(\text{OPT})}(y) \triangleq \arg\min_{\ell\in[1:M]} |y-\sqrt{\mathsf{S}}x_\ell|^2 - \ln I_0\big(2\sqrt{\mathsf{I}}|y-\sqrt{\mathsf{S}}x_\ell|\big), \tag{2}$$

where I_0 denotes the modified Bessel function of the first kind of order zero, which satisfies $I_0(z) \in [1, e^{|z|}]$ for $z \in \mathbb{R}$.

C. Low INR Regime

We showed in [6, eq.(4)] that for $\mathsf{I} \ll \mathsf{S}$, we can tightly approximate the optimal decoder (by using $I_0(z) \approx 1$ in (2))

$$\hat{\ell}^{(\text{OPT})}(y) \approx \hat{\ell}^{(\text{TIN})}(y) \triangleq \arg\min_{\ell\in[1:M]} |y-\sqrt{\mathsf{S}}x_\ell|^2, \tag{3}$$

where the superscript 'TIN' stands for Treat Interference as Noise. In other words, the optimal decoder is the minimum Euclidean distance decoder that is optimal for Gaussian noise only channels.

The TIN decoder in (3) provides the following upper bound on the SER of the ML decoder [6, eq.(14)-(16)]

$$P_\text{e}^{(\text{OPT})} \le P_\text{e}^{(\text{TIN})} = \frac{1}{M}\sum_{\ell\in[1:M]} \mathbb{E}_\Theta\left[Q\left(\sqrt{\frac{\mathsf{S}d_{\min}^2}{2}} - \sqrt{2\mathsf{I}}\cos(\Theta)\right)\right], \tag{4}$$

where we define the minimum distance as

$$d_{\min} := \min_{k\neq\ell} \ |x_k - x_\ell|. \tag{5}$$

D. High INR Regime

We showed in [6, eq.(5)] that for $\mathsf{I} \gg \mathsf{S}$, we can tightly approximate the optimal receiver (by using $I_0(z) \approx e^z$ in (2))

$$\hat{\ell}^{(\text{OPT})}(y) \approx \hat{\ell}^{(\text{IC})}(y) \triangleq \arg\min_{\ell\in[1:M]} \Big(\Re\{e^{-j\Theta}(y-\sqrt{\mathsf{I}}e^{j\Theta}-\sqrt{\mathsf{S}}x_\ell)\}\Big)^2. \tag{6}$$

With (6), we see that the radar interference can be subtracted off the received signal (indeed the superscript 'IC' stands for Interference Cancellation) but one of the two dimensions of the received signal is lost. In other words, the channel reduces to a real-valued phase-fading Gaussian channel.

The IC decoder in (6) provides the following upper bound on the SER of the ML decoder [6, eq.(18)-(20)]

$$P_e^{(\mathrm{OPT})} \le P_e^{(\mathrm{IC})} \overset{\mathsf{I}\gg\mathsf{S}\gg 1}{\approx} \frac{1}{M} \sum_{\ell\in[1:M]} \mathbb{E}_\Theta \left[Q\left(\Delta^+_{k,\ell}(\Theta)\right) + Q\left(\Delta^-_{k,\ell}(\Theta)\right) \right], \quad (7)$$

where $r_\ell := \Re\{e^{-j\Theta} x_\ell\}, \text{ for } \ell \in [1:M]$,

$$\Delta^+_{k,\ell}(\Theta) := \min_{k\neq\ell:\mathrm{sign}(r_k-r_\ell)>0} \sqrt{\frac{\mathsf{S}}{2}} |r_k - r_\ell|,$$

$$\Delta^-_{k,\ell}(\Theta) := \min_{k\neq\ell:\mathrm{sign}(r_k-r_\ell)<0} \sqrt{\frac{\mathsf{S}}{2}} |r_k - r_\ell|.$$

III. Constellation Design

We discuss the optimization algorithm for the constellation design in the low and high INR regimes.

A. Optimization Formulation

Our goal is to find the signal constellation such that the number of points M is maximized for a given power constraint and a given maximum SER. Mathematically, for some desired upper bound on the SER of ε, we aim to determine

$$M^{(\mathrm{OPT})}(\varepsilon) = \max \quad M \quad (8)$$

$$\text{s.t.} \quad P_e(\mathcal{X}) \le \varepsilon, \quad (9)$$

$$\mathcal{X} = \{x_1, \cdots, x_M\}, \quad (10)$$

$$\frac{1}{M}\sum_{\ell=1}^{M} |x_\ell|^2 \le 1, \; x_\ell \in \mathbb{C}. \quad (11)$$

where $P_e(\mathcal{X})$ denotes an upper bound to the optimal SER for the constellation $\mathcal{X}$, and is given by (4) and (7) at low and high INR, respectively. With (8), the location of the constellation in (10) has to be optimized such that the constraints for the average SER in (9) and average power in (11) are met. Note that such a constellation needs not necessarily exist as the SER has to be at least as large as that of a binary-PAM.

In our optimization algorithm, we start by fixing the number of points M (as small as 2) and find such a signal constellation that minimizes our SER approximations to that of the optimal decoder; if the SER for the found constellation satisfies ε then we increase M by 1 and repeat these steps until the maximum SER requirement is violated. We then obtain the constellation $\mathcal{X}$ with the largest number of points $M^{(\mathrm{OPT})}$ under the given constraints.

Our problem formulation is non-convex, thus we solved it by using the numerical Global Search (GS) method [8] which is available in the MATLAB Global Optimization Toolbox. Global Search is a gradient-based algorithm that uses a scatter-search mechanism to generate multiple randomized start points, then analyzes and rejects those points that are unlikely to improve the best local minimum found so far. Global Search aims to find the function's global minima; it attempts this by finding and comparing different local minima of smooth nonlinear optimization problem; because of this, the results are not always guaranteed to be globally optimal. In order to minimize the chance of having found a local optimum, we run the GS method multiple times with different starting points as well as other optimization parameters.

B. Optimization Results for the Low INR Regime

The designed signal constellations with maximum number of points $M^{(\mathrm{OPT})}$ for fixed ε of 10^{-3}, 10^{-5}, and 10^{-6} at $\mathsf{S}_{\mathrm{dB}} = 10, 15$, and 20 and $\mathsf{I}_{\mathrm{dB}} = \frac{\mathsf{S}_{\mathrm{dB}}}{4}$ are shown in Table I.

Based on the results, we observe that a triangle is initially formed with just 3 points, and then more points are added to form more triangles. At $\mathsf{S}_{\mathrm{dB}} = 20$ and $\varepsilon = 10^{-6}$, the designed constellation is hexagonal-like with $M = 7$ points, one of which is at zero. As we relax the SER requirements, which allows for the use of more points, we observe that the constellations have multiple concentric hexagonal layers; for example, as many as 12 points can be transmitted with the proposed shape to achieve $\varepsilon = 10^{-3}$. In general, as the number of points increases, the designed constellation tends to a hexagonal lattice. Our results are consistent with [4], in which it was shown that two-dimensional hexagonal-like signal constellations are optimal in complex-valued Gaussian noise channels at high SNR.

In Fig. 1(a), we show $M^{(\mathrm{OPT})}$ as a function of S_{dB}. These are the same constellations reported in Table I. As expected, $M^{(\mathrm{OPT})}$ generally increases with S; however, slightly relaxing the SER constraint for a specific SNR does not necessarily increase $M^{(\mathrm{OPT})}$ as adding just one more point may result in a violation of the maximum SER requirement.

In Fig. 1(b), we show the ML decoding regions of the signal constellations optimized for $\mathsf{S}_{\mathrm{dB}} = 20$, $\mathsf{I}_{\mathrm{dB}} = 5$, and $\varepsilon = 10^{-5}$ with 8 points from Table I, referred to as 8-GS-OPT-LOW. The bullet symbols (numbered from 1 to 8) indicate the position of the constellation points. The designed constellation is an almost-equilateral hexagon with a center point at the origin.

C. Optimization Results for the High INR Regime

The designed signal constellations with maximum number of points $M^{(\mathrm{OPT})}$ for fixed ε of $10^{-0.82}$, $10^{-1.15}$, and $10^{-1.48}$ at $\mathsf{S}_{\mathrm{dB}} = 10, 20$, and 30, and $\mathsf{I} \to \infty$ are shown in Table II. By graphically inspecting the $\log_{10}(P_e)$ versus S_{dB} for a classical M-PAM at high INR, where P_e is given by $2(1-\frac{1}{M})\mathbb{E}\left[Q\left(\sqrt{\frac{6}{M^2-1}\mathsf{S}\cos(\Theta)}\right)\right]$ from [6, eq(23)], we can conclude that a diversity order of $\frac{1}{2}$ is obtained at high SNR, that is, $\log_{10}(P_e) = -\frac{\mathsf{S}_{\mathrm{dB}}}{20} - \alpha$ with α being a constant that depends on M only. We also see that to impose $\log_{10}(\varepsilon) = -3$, a high SNR of at least 45 dB is required to transmit 2 points for an uncoded communication system. As a result, our designed constellations yield quite high error rates due to the low SNR range in our optimization examples.

Based on the results, we observe that the designed constellations are PAM-like but the points are not equally-spaced in general. The intuition is that the constellation points are placed as far apart from one another as possible (given the average power constraint) so as to result in larger possible minimum distances. Recall that the channel is equivalent to a real-valued phase-fading channel in this regime so it makes intuitive sense that the points are placed according to the optimal packing in

TABLE I: The optimal constellations at various values of S_{dB} and $\mathsf{I}_{\text{dB}} = 0.25\mathsf{S}_{\text{dB}}$ under certain SER constraints.

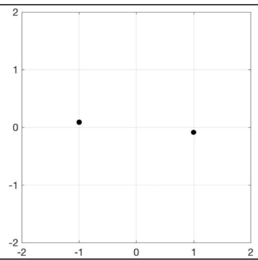
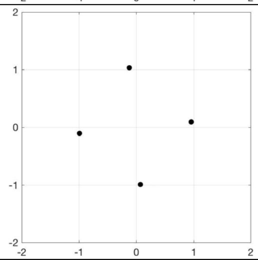
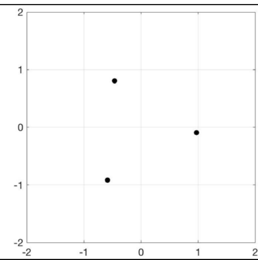
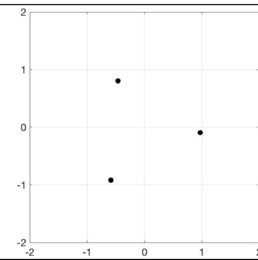
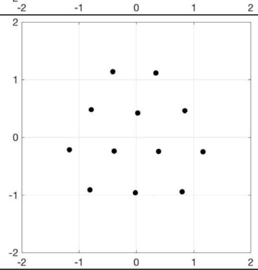
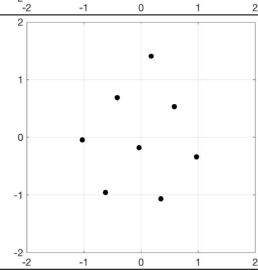
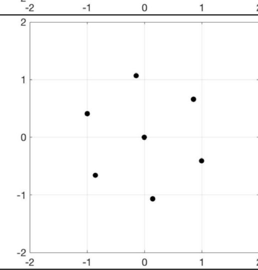
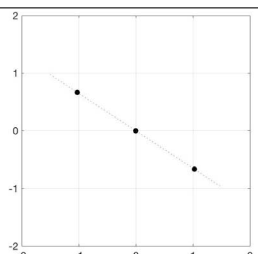
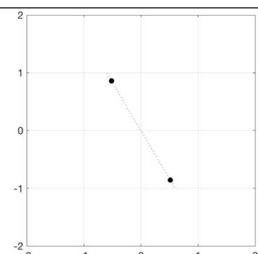
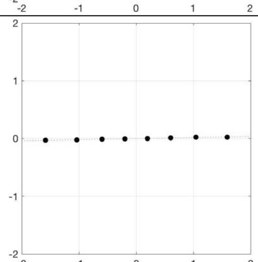
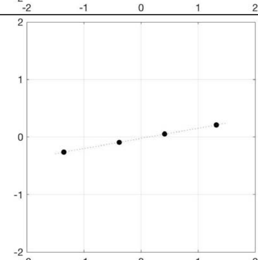
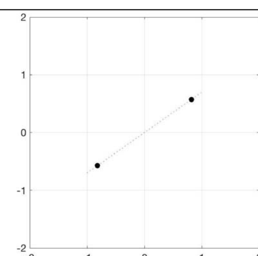
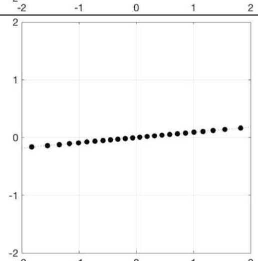
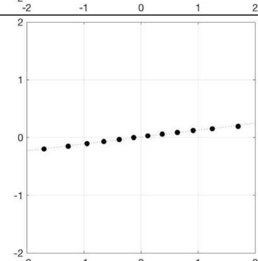
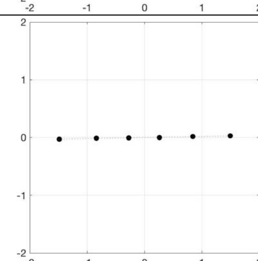
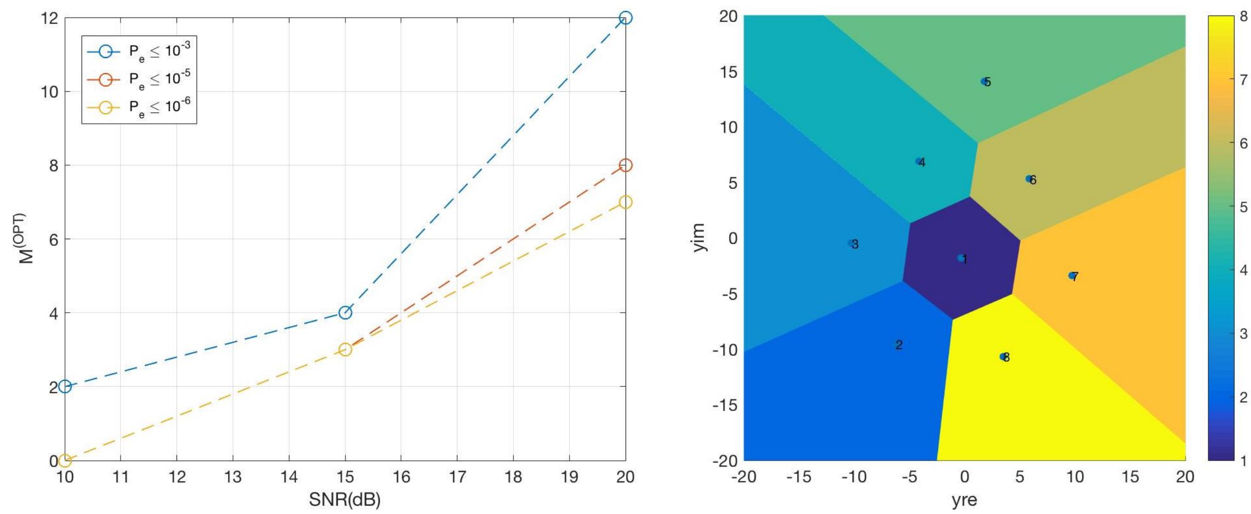
S_{dB} \ ε	10^{-3}	10^{-5}	10^{-6}
10		No constellations satisfying the fixed SER of 10^{-5} at $\mathsf{S} = 10$ dB are found.	No constellations satisfying the fixed SER of 10^{-6} at $\mathsf{S} = 10$ dB are found.
15			
20			

TABLE II: The optimal constellations at various values of S_{dB} and $\mathsf{I} \rightarrow \infty$ under certain SER constraints.

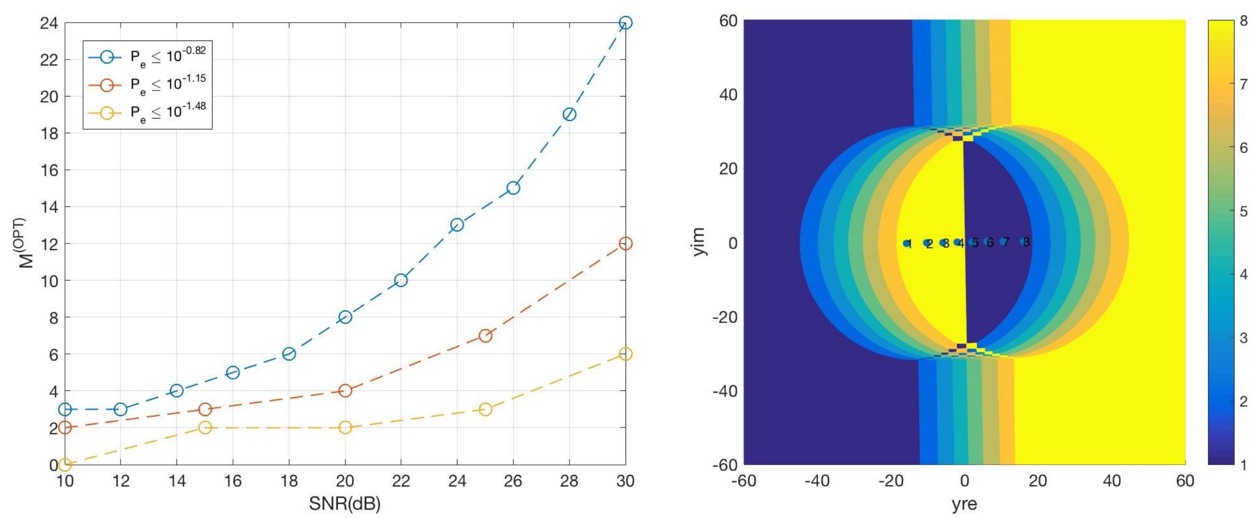
S_{dB} \ ε	$10^{-0.82}$	$10^{-1.15}$	$10^{-1.48}$
10			No constellations satisfying the fixed SER of $10^{-1.48}$ at $\mathsf{S} = 10$ dB are found.
20			
30			



(a) $M^{(\text{OPT})}$ vs. S_{dB} for fixed $\varepsilon = 10^{-3}, 10^{-5}$, and 10^{-6}.

(b) 8-GS-OPT-LOW decoding regions at $\mathsf{S} = 20$ dB and $\mathsf{I} = 5$ dB.

Fig. 1: Constellation Optimization at Low INR.



(a) $M^{(\text{OPT})}$ vs. S_{dB} for fixed $\varepsilon = 10^{-1.00}, 10^{-1.15}$, and $10^{-1.50}$.

(b) 8-GS-OPT-HIGH decoding regions at $\mathsf{S} = 20$ dB and $\mathsf{I} = 30$ dB.

Fig. 2: Constellation Optimization at High INR.

one dimension (at least at high SNR), which is the equi-lattice (equally-spaced points on a straight line).

In Fig. 2(a), we show $M^{(\text{OPT})}$ as a function of S_{dB} for the constellations reported in Table I with the addition of $M^{(\text{OPT})}$ at SNR increments of 2 dB for $\varepsilon = 10^{-0.82}$ and 5 dB for $\varepsilon = 10^{-1.15}$ and $10^{-1.48}$. Note that a small SNR increase does not necessarily increase $M^{(\text{OPT})}$ due to the SER constraint.

In Fig. 2(b), we show the ML decoding regions of the signal constellation optimized for $\mathsf{S}_{\text{dB}} = 20$, $\mathsf{I}_{\text{dB}} = 30$, and $\varepsilon = 10^{-0.82}$ with 8 points from Table II, referred to as 8-GS-OPT-HIGH. The bullet symbols (numbered from 1 to 8) indicate the position of the constellation points. The designed constellation is shaped somewhat like a PAM. Notice that the points are not equally spaced and the distances between the points start to increase as the points are positioned further away from the center point.

D. Comparison as a Function of INR

We report the SER as a function of INR in Fig. 3 for the constellations in Fig. 1(b) and 2(b), which were designed for $\mathsf{S}_{\text{dB}} = 20$. Note that those constellations were optimized for a fixed INR and thus may not be optimal for the whole INR range. For comparison, we also report the SER for the 8-PAM and 8-PSK, as representatives of the type of constellations that may be used in current practical systems. Practically, we should expect to transmit more than just 8 points. However, we picked 8 points for this numerical example as this was the

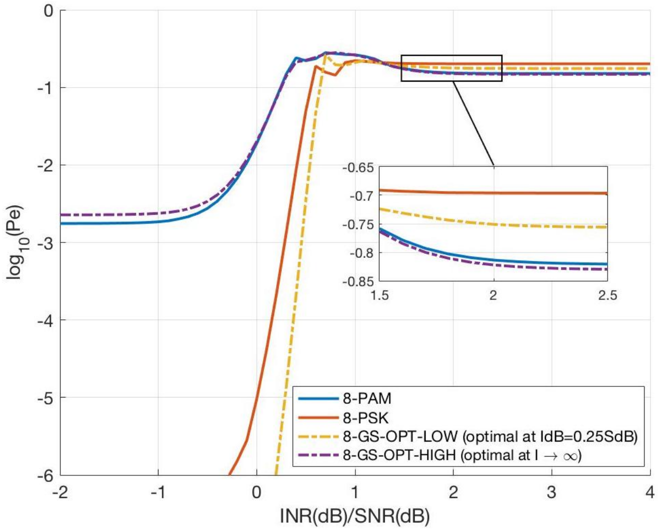


Fig. 3: SER Comparison for 8 points and $\mathsf{S} = 20$ dB.

TABLE III: Max M at various S_{dB} and I_{dB} for $\varepsilon = 10^{-1}$.

(a) PAM.

I_{dB} \ S_{dB}	10	15	20
$0.25\mathsf{S}_{\mathrm{dB}}$ (TIN)	$M^{(\mathrm{PAM})} = 3$ $P_{\mathrm{e}} = 10^{-1.2210}$	$M^{(\mathrm{PAM})} = 5$ $P_{\mathrm{e}} = 10^{-1.0241}$	$M^{(\mathrm{PAM})} = 7$ $P_{\mathrm{e}} = 10^{-1.3020}$
$+\infty$ (IC)	$M^{(\mathrm{PAM})} = 2$ $P_{\mathrm{e}} = 10^{-1.2377}$	$M^{(\mathrm{PAM})} = 3$ $P_{\mathrm{e}} = 10^{-1.1512}$	$M^{(\mathrm{PAM})} = 5$ $P_{\mathrm{e}} = 10^{-1.0838}$

(b) PSK.

I_{dB} \ S_{dB}	10	15	20
$0.25\mathsf{S}_{\mathrm{dB}}$ (TIN)	$M^{(\mathrm{PSK})} = 4$ $P_{\mathrm{e}} = 10^{-1.3706}$	$M^{(\mathrm{PSK})} = 8$ $P_{\mathrm{e}} = 10^{-1.0835}$	$M^{(\mathrm{PSK})} = 13$ $P_{\mathrm{e}} = 10^{-1.1219}$
$+\infty$ (IC)	$M^{(\mathrm{PSK})} = 2$ $P_{\mathrm{e}} = 10^{-1.2377}$	$M^{(\mathrm{PSK})} = 3$ $P_{\mathrm{e}} = 10^{-1.1275}$	$M^{(\mathrm{PSK})} = 5$ $P_{\mathrm{e}} = 10^{-1.0020}$

(c) OPT.

I_{dB} \ S_{dB}	10	15	20
$0.25\mathsf{S}_{\mathrm{dB}}$ (TIN)	$M^{(\mathrm{OPT})} = 4$ $P_{\mathrm{e}} = 10^{-1.3725}$	$M^{(\mathrm{OPT})} = 11$ $P_{\mathrm{e}} = 10^{-1.0301}$	$M^{(\mathrm{OPT})} = 26$ $P_{\mathrm{e}} = 10^{-1.0562}$
$+\infty$ (IC)	$M^{(\mathrm{OPT})} = 2$ $P_{\mathrm{e}} = 10^{-1.2377}$	$M^{(\mathrm{OPT})} = 3$ $P_{\mathrm{e}} = 10^{-1.1512}$	$M^{(\mathrm{OPT})} = 5$ $P_{\mathrm{e}} = 10^{-1.0882}$

same constellation size found by the previous constellation design from Table I and II at $\mathsf{S}_{\mathrm{dB}} = 20$.

For low INR, the classical 8-PAM and 8-PSK are markedly suboptimal; the best performance is attained by the constellation in Fig. 1(b) (which was optimized for $\mathsf{I}_{\mathrm{dB}}/\mathsf{S}_{\mathrm{dB}} = 1/4$, that is, $\mathsf{I}_{\mathrm{dB}} = 5$). At $\mathsf{I} = 0$ (not reported in Fig. 3), the suboptimal 8-PSK yields $\log_{10}(P_{\mathrm{e}}) = -7.21$ while our designed constellation 8-GS-OPT-LOW achieves the lowest error rate of $\log_{10}(P_{\mathrm{e}}) = -10.82$. For high INR, the classical 8-PAM is competitive with the constellation in Fig. 2(b) (which was optimized for $\mathsf{I} \to \infty$).

At all range of INR, we remark that the designed constellations outperform practically-used ones in terms of SER.

E. Corresponding Transmission Rates for a Fixed SER.

Finally, Table III compares the largest rate of the designed constellations with the practical ones, which are PAM and PSK in our examples, subject to the average SER constraint of $\varepsilon = 10^{-1}$ at $\mathsf{S}_{\mathrm{dB}} = 10, 15$, and 20. At low INR ($\mathsf{I}_{\mathrm{dB}} = 0.25\mathsf{S}_{\mathrm{dB}}$), the designed constellations achieve the largest rates compared to PSK and PAM. As the radar interference increases, less points can be sent. At high INR ($\mathsf{I}_{\mathrm{dB}} = 2\mathsf{S}_{\mathrm{dB}}$), PSK and PAM are very competitive with the designed constellations (due to the low SNR range in our examples). In these cases, the designed constellations yield lower error rates than the classical ones.

IV. Conclusions

In this paper, we obtained the two-dimensional signal constellation for uncoded transmission that has the largest number of points (rate), subject to an average input power constraint and a quality-of-service constraint in terms of maximum allowed average symbol error rate, for a complex-valued channel impaired by additive Gaussian noise and radar interference. We thoroughly investigated two regimes: weak and very strong radar interference, and concluded that the designed constellation tends to have a hexagonal-like shape at low INR while it tends to have a PAM-like shape at high INR and that it outperforms the commonly-used ones.

Future work includes the constellation design in the mid INR regime.

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